Estimation of Heat Sources in Nuclear Reactors

LLOYD G. ALEXANDER

Oak Ridge National Laboratory, Oak Ridge, Tennessee

Knowledge of local heating rates is needed for estimation of operating temperatures, thermal stresses, and cooling requirements in nuclear reactor components. Heat is liberated by the dissipation of the energy of fission fragments, beta particles, fast neutrons, and gamma photons. Heating rates are formulated in terms of either neutron or gamma flux densities, the corresponding collision probabilities, and appropriate energy transfer coefficients, the forms of which are given. Special methods of estimating the flux densities are discussed. The data on the magnitudes of the various energy sources are reviewed.

The designer of nuclear reactors needs to know the local rates of heat liberation in the various reactor components in order to estimate temperatures, cooling requirements, thermal stresses, corrosion rates, radiation-damage annealing rates, and temperature-dependent reactivity changes. The heat-liberation rates can be formulated in terms of the local neutron and gamma fluxes, ϕ_n and ϕ_γ. Admittedly, values of these are not easily come by; nevertheless, recent progress in neutron- and gamma-particle transport theories makes it possible to treat many systems of interest to engineers. It is the purpose of this paper to formulate the heat-liberation rates and to indicate methods of estimating the fluxes.

ENERGY SOURCES AND MODES OF LOCAL DEPOSITION OF HEAT

Heat is liberated in reactors by the dissipation of the kinetic energy of fission fragments, beta particles, gamma photons, and neutrons. These particles, in penetrating matter, collide with electrons and atomic nuclei and impart energy to them. In some collision processes part of this energy is reradiated, but for the most part it is rather rapidly degraded into thermal motion of nuclei and electrons in the macroscopically near vicinity of the collision.

Fission fragments, primary gamma photons, and neutrons result directly from fission. Fission fragments emit beta particles and gamma photons by radioactive decay. Neutrons diffuse through the reactor and generate gamma photons by inelastic collisions with and absorptions in various nuclei. Photons arising from all these processes may experience many scattering collisions, losing all or part of their energy at each collision.

Fission-fragment and Beta-decay Energy

According to Shapiro (*18*), two and occasionally three heavy particles result from the fission of U^{235} by thermal neutrons. Hanna (*11*) critically examined the data as of 1950 and concluded that the most probable value for the average kinetic energy of the fission fragments E_{ff} lies in the range 168 ± 5 mev./fission.

Because of their high charge, fission fragments have short ranges (*18*). Accordingly, from a macroscopic point of view their energy is dissipated at the point of fission.

Way (*23*) estimated the energy released by the beta decay of fission fragments $E_{ff\beta}$ to be 7 mev./fission. Effective ranges are of the order of 1 mm. or less in reactor materials; hence, the beta energy is also liberated very near the point of fission.

The sum E_f of the fission-fragment and beta-ray energies is thus about 175 mev./fission. The heat-release rate is proportional to the number of fissions per unit volume per unit time and may be formulated as follows:

$$(G_{ff} + G_{ff\beta}) = \int_{E_n=0}^{E_n=\infty} \Sigma_f(E_n)\phi_n(E_n)E_f \, dE_n \quad (1)$$

Here $\phi_n(E_n)$ represents the energy-dependent neutron flux density, i.e., total path length (cm.) traversed by neutrons having energy E_n (mev.) per unit volume (cc.) per unit time (sec.) per unit energy range (mev.), and $\Sigma_f(E_n)$ is the probability, per centimeter of travel, that a neutron of energy E_n will experience a collision resulting in fission. The integration is carried over all energies; however, there are negligibly few neutrons having energies greater than 7 mev. In thermal reactors the great majority of the flux is concentrated into a narrow band around the temperature of the medium. Furthermore, the fission cross section $\Sigma_f(E_n)$ decreases rapidly with increasing neutron energy. Accordingly, in thermal reactors the integral is approximated very well by the product of the thermal flux (say, the flux lying between 0.0 and 0.1 ev.) and the fission cross section evaluated at the temperature of the medium. The thermal flux may be denoted by $\phi_n(th)$, whence

$$\phi_n(th) = \int_{0.0}^{0.1\text{ ev}} \phi_n(E_n)\, dE_n \quad (2)$$

Using Equation (2) and converting to B.t.u./(hr.)(cu. ft.) gives Equation (1) in the form, for thermal reactors,

$$G_f = (G_{ff} + G_{ff\beta}) \frac{\text{B.t.u.}}{\text{(hr.)(cu. ft.)}} = 2.70 \times 10^{-6}\Sigma_f(th)\phi_n(th) \quad (3)$$

Neutron Energy

Three processes by which neutrons impart kinetic energy to atomic nuclei are (*a*) absorption, (*b*) elastic scattering, and (*c*) inelastic scattering.

In an absorption collision the neutron enters and combines with the target nucleus N, forming the next higher isotope N'. The nucleus is "knocked on,"

and its velocity is readily calculated from a simple momentum balance.

Defining δ_{nc} as the average fraction of the kinetic energy of the incident neutron appearing as kinetic energy of the target nucleus after a capture collision, one has

$$\delta_{nc} = \overline{E_{N'}/E_n} = m/(M + m) \quad (4)$$

where m denotes the atomic mass of the neutron, M the atomic mass of the target nucleus, and $\overline{E_{N'}}$ the average postcollision kinetic energy of the nucleus.

The remainder of the neutron kinetic energy plus the binding energy is released in the form of a series of so-called "capture" gamma photons of decreasing energy. The N' isotope may be radioactive and may decay by beta-gamma emission. Later sections deal with heat release from these sources.

The local heating effect due to the capture of fast neutrons is thus

$$G_{nc} = \int_{E_0}^{\infty} \delta_{nc} E_n \Sigma_{nc}(E_n) \phi_n(E_n)\, dE_n \quad (5)$$

where E_n denotes the energy of the neutrons and where Σ_{nc} is the so-called "macroscopic" neutron-capture cross section. It is the probability that a neutron will experience a capture collision per unit length of travel. The lower limit of integration, E_0, is chosen sufficiently high so that Equation (5), derived on the assumption that the energy of the target nucleus prior to the collision is negligible in comparison with the initial neutron energy, remains valid but also sufficiently low so that only a negligible portion of the neutron energy remains to be transferred at energies less than E_0.

The transfer coefficient δ_{nc} of Equation (4) is small except in materials having low atomic mass. Moderator materials have low atomic mass (12 or less), but they are selected, for reasons of neutron economy, from among materials having low capture cross sections in comparison with their scattering cross sections. For all materials the capture cross sections tend to become small at high neutron energies, while at low neutron energies, where the cross sections may be appreciable, the energy available for transfer is small. Thus it turns out that the contribution G_{nc} of neutron capture to the local heating rate is always small compared with the heat released by neutron elastic scattering and the dissipation of gamma energy. Neutron captures cannot be neglected, however, because of the accompanying emission of high-energy "capture" gammas, which will be considered in a later section.

In elastic collisions the neutron does not react with the nucleus, and both momentum and kinetic energy are conserved. The appropriate balances are most conveniently formulated in the center-of-mass system of coordinates. By combining these and transforming the results to the laboratory system of coordinates, Glasstone and Edlund (*7*) obtained a result that may be put into the form

$$E_n'/E_n = \frac{M^2 + 2Mm\cos\theta + m^2}{(M + m)^2}$$

where E_n' is the postcollision neutron energy and θ is the angle of scattering in the center-of-mass system. Since kinetic energy is conserved in the collision, it follows that

$$E_N/E_n = 1 - E_n'/E_n$$

where E_N is the postcollision energy of the target nucleus.

The scattering is usually isotropic in the center-of-mass system; hence the probability $P(\theta)\, d\theta$ that a neutron will be scattered elastically through an angle θ into angle $d\theta$ is related (*7*) to the angle θ by

$$P(\theta)\, d\theta = \sin\theta\, d\theta/2.$$

By definition the average fraction δ_{ne} of neutron energy E_n converted into kinetic energy E_N of the target nucleus is given by the integral

$$\delta_{ne} = \int_0^{\pi} (E_N/E_n) P(\theta)\, d\theta$$

which leads to the simple result that

$$\delta_{ne} = (1 - \alpha)/2 \quad (6)$$

where

$$\alpha = [(M - m)/(M + m)]^2$$

The elastic-collision heating rate is formulated similarly to Equation (5):

$$G_{ne} = \int_{E_0}^{\infty} \delta_{ne} E_n \Sigma_{ne}(E_n) \phi_n(E_n)\, dE_n \quad (7)$$

The lower limit E_0 may be taken at about 0.01 mev. G_{ne} must be summed with respect to all kinds of atoms present in the medium.

If the compound nucleus formed by the interaction of a neutron with a target nucleus decomposes by the emission of another neutron, the net result is equivalent to the scattering of the incident neutron. The nucleus however is left in an excited state, kinetic energy is not conserved, and the scattering therefore is inelastic.

The neutron energy may be reduced severalfold as a result of the collision. The major portion of this energy loss appears as energy of excitation of the nucleus and is rather quickly radiated in the form of "inelastic-scattering" gammas. The kinetic energy acquired by the nucleus is generally less than that which it would have received by capture of the neutron. This kinetic energy, which is the only part of the energy transferred that appears locally as heat, is small in comparison with that released by dissipation of gamma energy for the same reasons that the corresponding contribution from neutron capture is small, except that the cross sections for inelastic scattering may be larger than for capture. At present writing it is not clear that the local contribution to the heating rate of the inelastic-scattering process is always negligible. In any event, the process cannot be neglected because of the accompanying emission of gamma rays, which sometimes makes the single most important contribution to the local heating rate (*26*). This aspect of the process is considered further in a later section.

Energy Release by Beta Decay of Activated Nuclei and Related Processes

Nuclei rendered unstable by the capture of a neutron may emit alpha, beta, or other particles as well as gamma photons. Beta emission is the most common mode of decay. For a given transition the beta particles are emitted with energies ranging continuously from zero up to some characteristic maximum $E_\beta(\max)$ which is the most commonly tabulated value (*12, 14*). As the contribution to the local heating from beta decay of activated nuclei is usually small compared with other contributions, it is generally sufficiently precise (*12*) to take the average electron energy to be $\frac{1}{3}\, E_\beta(\max)$. Thus the local heating rate due to "capture" betas becomes

$$G_{nc\beta} = \int_0^{\infty} \Sigma_{nc}(E_n)\phi_n(E_n)\, dE_n \cdot \sum_i N_i E_{\beta i}(\max)/3 \quad (8)$$

where N_i is the number of betas of characteristic energy $E_{\beta i}(\max)$ emitted per neutron capture in the decay chain involving i groups of betas. The decay process may proceed through several isomeric states and atomic numbers. It should be noted that the contribution from "thermal" neutrons in Equation (8) may be large and override the contribution from fast neutrons.

The contribution of beta decay to the local heating rate is negligible in all moderators except possibly light water. It may become appreciable in structural materials, fuel diluents and clad, coolants, and other materials exposed to high thermal neutron fluxes when these have appreciable capture cross sections, as in stainless steels.

The isotopes Li^6 and B^{10} emit alpha particles following capture of neutrons. As the capture cross sections are large and as nearly all the kinetic energy of the incident neutron plus the binding energy appears as kinetic energy of the product nucleus (there being only a weak capture gamma ray emitted by B, none by Li), the local heating may become important if the concentrations of these nuclides become

appreciable. The effect is computed by means of Equation (8), but the summation is replaced by the sum of E_n and the energy equivalent of the mass change, which for Li^6 amounts to 4.78 mev. and for B^{10} to 2.31 mev.

Gamma Heating

Gamma photons penetrating matter exhibit three interaction processes: (*a*) photoelectric effect, (*b*) pair production, and (*c*) Compton scattering.

In the first process a photon transfers all its energy to an electron, which is "knocked on" and dissipates its energy in the immediate vicinity. The probability that a photon will experience such a collision per unit length of travel is denoted by $\mu_{pe}(E_\gamma)$. When the atom from which the electron was ejected acquires another electron in its place, an X ray of low energy is emitted; however, about 95% of these are reabsorbed inside the atom, and an Auger electron of low energy is ejected from among the outer valance electrons. This electron also dissipates its energy in the vicinity. Various other energy-"dribbling" processes may occur, but the net effect is that all the energy of the primary gamma photon is dissipated as heat in the vicinity of the collision. Thus the fraction $\delta_{\gamma pe}$ of initial gamma energy E_γ dissipated is unity. Consequently the heating rate due to photoelectric collisions of gamma photons is given by

$$G_{\gamma pe} = \int_0^\infty E_\gamma \mu_{pe}(E_\gamma)\phi_\gamma(E_\gamma)\, dE_\gamma \quad (9)$$

where E_γ denotes the energy of the gamma photon and $\phi_\gamma(E_\gamma)$ denotes the energy-dependent gamma flux.

Photons having energies in excess of 1.02 mev. may interact with the coulomb field of a nucleus and produce an "electron pair." The probability of pair production per unit length of travel is denoted by $\mu_{pp}(E_\gamma)$. The resulting particles dissipate their kinetic energy near the point of production. The positron ultimately combines with an electron forming two photons having energies approximately 0.51 mev. each. These may penetrate some distance from the point where the pair production occurred. The nucleus involved in the collision also acquires some energy. The net result is that $(E_\gamma - 1.02)$ mev. of energy is dissipated at the point of collision. Thus $\delta_{\gamma pp}$, the fraction of photon energy E_γ dissipated locally, becomes

$$\delta_{\gamma pp} = (E_\gamma - 1.02)/E_\gamma; \qquad E_\gamma \geq 1.02 \quad (10)$$

The heating rate due to pair production takes the form

$$G_{\gamma pp} = \int_0^\infty \delta_{\gamma pp} E_\gamma \mu_{pp}(E_\gamma)\phi_\gamma(E_\gamma)\, dE_\gamma \quad (11)$$

The lower limit is taken as zero for later convenience, but $\mu_{pp}(E_\gamma)$ is zero for E_γ less than 1.02 mev.

Compton scattering is the scattering resulting from an elastic collision of a photon with an electron. For this process Compton derived the following relation between the scattering angle θ and the energies E_γ and E_γ' of the photon before and after collision:

$$E_\gamma'/E_\gamma = \frac{1}{1 + E_\gamma(1 - \cos\theta)/(mc^2)} \quad (12)$$

Since the collision is elastic, kinetic energy is conserved, and

$$\frac{E_\beta}{E_\gamma} = \frac{E_\gamma - E_\gamma'}{E_\gamma} = 1 - \frac{E_\gamma'}{E_\gamma} \quad (13)$$

where E_β is the energy of the electron. The probability that a photon will be scattered through an angle θ into angle $d\theta$ is given by the Klein-Nishina formula (*10*), which may be put into the form

$$P(\theta)\, d\theta = K(E_\gamma'/E_\gamma)^2[(E_\gamma/E_\gamma') + (E_\gamma'/E_\gamma) - \sin^2\theta]\sin\theta\, d\theta \quad (14)$$

where K is a normalization constant such that

$$\int_0^\pi P(\theta)\, d\theta = 1$$

The fraction $\delta\gamma_c(E\gamma)$ of the energy of the incident photon that appears as kinetic energy E_β of the beta particle is given by

$$\delta_{\gamma c}(E_\gamma) = \int_0^\pi (1 - E_\gamma'/E_\gamma)P(\theta)\, d\theta \quad (15)$$

The integral has not been reduced to a convenient algebraic form but has been evaluated. (*See* Equations (18) ff.).

The energy deposition due to Compton scattering may be formulated as follows:

$$G_{\gamma c} = \int_0^\infty \delta_{\gamma c}(E_\gamma)\mu_c(E_\gamma)E_\gamma\phi_\gamma(E_\gamma)\, dE_\gamma \quad (16)$$

Summing the gamma heating contributions given by Equations (9), (11), and (16) gives

$$G_\gamma = \int_0^\infty (\delta\gamma_{pe}\mu_{pe} + \delta_{\gamma pp}\mu_{pp} + \delta_{\gamma c}\mu_c)E_\gamma\phi(E_\gamma)\, dE_\gamma \quad (17)$$

where the E_γ's have been omitted in the parenthesis for brevity. Substituting from Equation (10) and rearranging, while keeping in mind that μ_{pp} vanishes for energies less than 1.02 mev. and that μ_{pe} is unity, one finds

$$G_\gamma = \int_0^\infty \mu_e(E_\gamma)E_\gamma\phi_\gamma(E_\gamma)\, dE_\gamma - \int_{1.02}^\infty 1.02\mu_{pp}\phi_\gamma(E_\gamma)\, dE_\gamma \quad (18)$$

where

$$\mu_e(E_\gamma) = \mu_{pe} + \mu_{pp} + \delta_{\gamma c}\mu_c \quad (19)$$

The quantity $\mu_e(E_\gamma)$ appearing in the left-hand integral, divided by the density of the medium, has been graphed by Snyder and Powell (*20*) as a function of E_γ for various materials. Their coefficient $\mu - \sigma_s$ is identical with $\mu_e(E_\gamma)$. It is called the energy absorption coefficient. If the so-called "annihilation photons," resulting from the neutralization of the positron, are assumed to be absorbed in the vicinity of the point of pair production, one may neglect the right-hand integral provided the annihilation photons are not included in the gamma flux $\phi_\gamma(E_\gamma)$.

NEUTRON FLUX DISTRIBUTION

The neutron diffusion and transport theories yield descriptions of the spectral and spatial distributions of the neutron fluxes $\phi_n(E_n)$. These theories have been discussed elsewhere (*7*) and will not be considered here in detail. A one- or two-group diffusion calculation will suffice to describe the "gross" variation of the thermal flux in many thermal reactors with sufficient precision for calculations of G_f. If $\phi_n(th, o)$ is the flux at some reference point in a homogeneous reactor (say the center) and $f(r)$ denotes a function, obtained from diffusion theory, describing the spatial variation of $\phi_n(th, r)$ where r denotes the radius vector from the reference point, such that

$$\phi_n(th, r) = \phi_n(th, o)f(r) \quad (20)$$

by Equation (3) it follows [by use of the notation of Equation (20)] that

$$G_f(r) = A\phi_n(th, o)f(r)\Sigma_f(th, r) \quad (21)$$

where A is a proportionality factor. The heat output from the reactor due to fission fragments Q_f is obtained by integrating $G_f(r)$ over the volume of the fuel-bearing part of the core.

TABLE 1

Geometry	$f(r)$	G_0/G_{avg}
Sphere of radius R	$\sin(\pi\rho/R)/(\pi r/R)$	3.29
Cube of side A	$\cos(\pi x/A)\cos(\pi y/A)\cos(\pi z/A)$	3.64
Right cylinder of radius R	$J_0(2.405\rho/R)\cos(\pi z/2R)$	3.87

$$Q_f = \int_{V_f} G_f(r)\, dV$$

where V_f denotes the volume of the fuel region. Substituting from Equation (21) and solving for $A\phi_n(th, o)$ gives

$$A\phi_n(th, o) = \frac{Q_f}{\int_{V_f} f(r)\Sigma_f(th, r)\, dV}$$

Replacing this in Equation (21) gives

$$G_f(r) = \frac{Q_f f(r)\Sigma_f(th, r)}{\int_{V_f} f(r)\Sigma_f(th, r)\, dV} \quad (22)$$

The energy available from fission fragments (kinetic and beta decay) is about 175 mev./fission. The total energy per fission is about 200 mev. Thus Q_f is about 87% of the total reactor heat output, Q_t. The balance of Q_t is generated by fission gammas (7.5 mev.), fission-fragment-decay gammas (6 mev.), fast neutrons (5 mev.), capture gammas and betas (7 mev.). The particles bearing this energy (about 25 mev.) tend to leak out of the fuel-bearing zones and to liberate heat in the coolant, reflector, thermal shield, pressure shell, and biological shield. However, in the designing of the fuel-bearing core, it is sufficient and conservative to assume that all the reactor heat Q_t of a reactor is realeased in the fuel and that it has the same spatial distribution as Q_f.

The function $f(r)$ has been evaluated by Glasstone and Edlund (7) for certain bare, homogeneous, thermal reactors of simple shape, with $\Sigma_f(th, r)$ uniform.

Integrating the given functions over the corresponding geometric forms readily yields the ratio of the maximum heat-release rate G_0 to the average rate G_{avg}. Values are listed in Table 1. The presence of moderating reflectors, control elements, structural materials, coolants, ducts, and nonuniform loading (variable Σ_f) modify these simple distributions. Space does not permit treatment of these effects, but it is important to reduce G_0/G_{avg} so that a larger fraction of the core may operate at or near the limiting conditions and so that burn-up of nuclear fuel and accompanying effects will be more uniform.

Glasstone and Edlund point out that the functions $J_0(x)$ and $(\sin x)/x$ are rather similar to $\cos x$ and may to a good first approximation be replaced by $\cos x$.

In order to increase resonance escape in low-enrichment uranium reactors, the fuel is sometimes segregated from the moderator. Fast neutrons, released by fission in the fuel, have a high probability of escaping into the moderator, where they are quickly slowed down without exposure to resonance absorption in U^{238}. In the O.R.N.L. graphite reactor this result is achieved by use of metallic fuel rods about 1 in. in diameter arranged in an 8-in. square array in a matrix of graphite. The fuel has negligible moderating properties; neutrons are "thermalized" almost entirely in the graphite and diffuse thence back into the fuel, there causing further fissions. As a result, there is a local gradient in the flux density both in the moderator and in the fuel superimposed on the gross flux distribution of Equation (20). Some reactors, which are heterogeneous from a phase standpoint, such as the M.T.R., are really homogeneous from a nuclear standpoint, there being negligible local gradients in the neutron flux in the moderator and fuel.

Neutron-diffusion theory has been employed in the calculation of thermal flux distributions in fuel elements in heterogeneous reactors (7). The basic equation is

$$D\nabla^2\phi_n(th, \rho) - \Sigma_a\phi_n(th, \rho) + S_n(th, \rho) = \partial n/\partial t$$

where n is the number of thermal neutrons per cubic centimeter, S_n is the volume source of thermal neutrons (due to slowing down of fast neutrons) in neutrons per cubic centimeter per second, D is the neutron-diffusion coefficient, Σ_a is the neutron-absorption coefficient (the so-called "macroscopic" absorption cross section), and t denotes time. In the steady state $\partial n/\partial t$ is zero; in the fuel S_n is zero. The solution, in polar coordinates (corresponding to the case of an infinitely long fuel rod immersed in a uniform external flux), is

$$\phi_n(th, \rho) = BI_0[\rho(\Sigma_a/D)^{1/2}] \quad (23)$$

where ρ is the distance from the axis of the fuel rod and B is an arbitrary constant whose magnitude is proportional to the power level at which the reactor is operating and which may be evaluated by multiplying the gross flux distribution, $f(r)$ of Equation (20), by the local variation $\phi_n(th, \rho)$ and proceeding through the operations indicated in Equation (22) to evaluate the product $AB\phi_n(th, o)$ in terms of Q_f.

Using Equation (23), Alexander (2) calculated the temperature distribution in fuel rods and plates and compared the results with temperature distributions calculated from flux distributions measured experimentally by Woods and Biehl (24). As expected, the agreement is only fair, since diffusion theory applies poorly in cases where the neutron mean free path is appreciable compared with the dimensions of the system being studied.

In the design of fuel elements it is customary and conservative practice to assume that all the fission energy (200 mev./fission) is released in the fuel. In estimating heat release in moderator, coolant, pressure shell, and shield, however, it is necessary to estimate the leakage out of the fuel zone of energy associated with fast neutrons and gamma rays. A fair estimate of the fast-neutron spectral distribution as a function of position in the moderator of heterogeneous reactors employing long fuel rods may be had from the "Fermi age" treatment (7) of simple models, e.g., an infinite matrix of line sources with negligible capture of fast neutrons in U^{235} and U^{238} and only weak capture in the moderator. The flux at radius ρ from a single infinite line source of strength S_n nascent neutrons per unit length per unit time immersed in infinite moderator is given by

$$\phi_n(\rho, E_n) = \frac{1}{E_n\xi\Sigma_t} \cdot \int_{E_s=E_n}^{\infty} \frac{S_n P(E_s)p(E_s, E_n)\, dE_s}{4\pi\tau(E_s, E_n)} \cdot \exp\left[-\rho^2/4\tau(E_s, E_n)\right] \quad (24)$$

where E_s is the energy of the nascent neutrons, $P(E_s)$ their spectral distribution, $p(E_s, E_n)$ the capture-escape probability, and $\tau(E_s, E_n)$ the Fermi age.

$$P(E_s) = 0.484(\sinh\sqrt{2E_s})\exp(-E_s)$$

$$p(E_s, E_n) = \exp\left(-\int_{E_n}^{E_s} \Sigma_a\, dE/\xi\Sigma_t E\right)$$

$$\tau(E_s, E_n) = \int_{E_n}^{E_s} D\, dE/\xi\Sigma_t E$$

Here ξ is the average logarithmic energy decrement of neutrons per collision; D is the diffusion coefficient; Σ_a and Σ_t are the absorption and total cross sections respectively.

The flux at any point in the moderator is constructed by summing the contributions given by Equation (24) for all line sources.

Alternatively the Monte Carlo technique (17) permits treatment of the spatial variation of the fast flux for the case of strong absorption in fuel, moderator, control elements and leakage out of the reactor, provided the fission-rate density in the fuel can be estimated by some means, e.g., by a multigroup diffusion calculation on a "homogenized" model of the core. In the Monte Carlo method the events in a neutron "life history" are selected by random sampling from an appropriate distribution. Initial neutron energies are selected from the fission-neutron spectral distribution, a flight direction is established, the coordinates of the first collision are found, kind of nucleus and type of collision are selected, postcollision neutron energy and new flight direction are determined. The history is followed until the neutron is absorbed or escapes from the reactor. The process is repeated until a sufficiently large number of cases has been obtained to permit a statistical analysis of the spatial and spectral distributions of the fast flux.

The technique is applicable to the calculation of fast fluxes in moderator, core vessels, shields, reflectors, thermal

shields, pressure shells, biological shields, etc. However, unless simplifications are introduced, especially geometric symmetries, the number of histories required may be excessive even for an electronic computer.

GAMMA-RAY-FLUX-DENSITY DISTRIBUTIONS

Compton scattering greatly complicates the estimation of gamma flux densities. The process is not isotropic and the energy decrement of the scattered photon is strongly dependent on the scattering angle. Photons scattered through small angles retain most of their original energy and may penetrate to remote regions of the reactor. Photons scattered through large angles suffer large energy losses, and their penetrating power is greatly reduced.

The important primary gamma sources are the fission process, the decay of fission fragments, neutron captures, inelastic-scattering collisions of neutrons. Compton scattering is the most important secondary source. At many points of interest, indeed at most, the contribution to the gamma flux made by scattered photons is many times greater than that of unscattered photons.

In principle, the gamma flux may be constructed by integration over the gamma sources. The vector coordinates of a point in the reactor may be denoted by r and the gamma flux density in photon centimeters per second per cubic centimeter per mev. by $\phi_\gamma(E_\gamma, r)$, r' denotes another point in the reactor, a volume increment about r' is denoted by dV_γ', and the total gamma source strength in photons per second per centimeter at r' is denoted by $S_\gamma(r')$. $P(E_\gamma, r')\ dV_\gamma'$ may be considered the conditional probability that a photon will be emitted from, or scattered out of, dV_γ' per mev. with energy E_γ. $P(r', r)$ may be considered the probability that the photon will be emitted per steradian in the direction of r. The probability that a photon per centimeter of travel will be removed from the beam, denoted by $\mu_t(E_\gamma)$, is simply the sum of the probabilities of the photoelectric, pair-production, and Compton-scattering collisions.

$$\mu_t(E_\gamma) = \mu_{pe}(E_\gamma) + \mu_{pp}(E_\gamma) + \mu_c(E_\gamma) \quad (25)$$

The probability $P(r' \rightarrow r)$ that a photon will arrive uncollided at r is thus simply (in homogeneous, isotropic media)

$$P(r' \rightarrow r) = \exp\left[-\mu_t(E_\gamma)(|\, r' - r\,|)\right]$$

where $|r' - r|$ denotes the scalar distance between r' and r. It follows that

$$\phi_\gamma(E_\gamma, r) = \int_{V_\gamma} S_\gamma(r') P(E_\gamma, r')\ dV_\gamma' \cdot P(r', r) \exp\left[-\mu_t(E_\gamma)(|\, r' - r\,|)\right] \quad (26)$$

The primary gamma sources are isotropic, and for these $P(r', r)$ has the value of $\frac{1}{4\pi}$ per steradian. In the case of the scattered-gamma source, $P(r', r)$ is a complicated function of the "directed" gamma flux $\phi_\gamma(E_\gamma, \vec{\Omega}, r')$, which is defined as the gamma flux at r' contributed by photons of energy E_γ moving in the direction $\vec{\Omega}$. It has units of photon-centimeter per second per cubic centimeter per mev. per steradian. Thus the construction of the flux $\phi_\gamma(E_\gamma, r)$ at r requires a knowledge of the more complicated flux $\phi_\gamma(E_\gamma, \vec{\Omega}, r')$ everywhere else. Consequently, a general formulation of $\phi_\gamma(E_\gamma, r)$ does not seem feasible.

The Monte Carlo technique may be applied to this problem (*17*). Suppose the histories of a sufficient number of primary photons of energy E_γ' are determined to define $F(E_\gamma'; r' \rightarrow r)$ identified as the average fraction of the energy E_γ' of photons originating at r' that is dissipated at r per cubic centimeter. The heating rate is now readily formulated as follows:

$$G_\gamma = \int_{V_\gamma} \int_{E_\gamma'=0}^{\infty} S_{p\gamma}(r')\ dV_\gamma' \cdot P(E_\gamma')\ dE_\gamma'\ E_\gamma' F(E_\gamma'; r' \rightarrow r) \quad (27)$$

where $S_{p\gamma}(r')$ is the primary gamma-source strength.

In principle, $F(E_\gamma'; r' \rightarrow r)$ may be evaluated for any case howsoever complicated in geometry or heterogeneity, but the sheer magnitude of the computational labor may render the method impractical. The simplest case is that of an infinite, homogeneous, isotropic medium. Life histories are generated in the usual manner and an account is kept of the amount of heat liberated and of the radial distance R of each collision from the point of origin. From these data it is easy to calculate the fraction $f(E_\gamma', R)\ dR$ of the energy E' of the original photons that is released inside a spherical annulus of radius R and thickness dR. The volume of such an annulus is $4\pi R^2\ dR$, from which it follows that $F(E_\gamma'; r' \rightarrow r)$ is given by

$$F(E_\gamma'; r' \rightarrow r) = \frac{f(E_\gamma', |\, r' - r\,|)}{4\pi(|\, r' - r\,|)^2} \quad (28)$$

A general solution of the foregoing case has been obtained by Goldstein and Wilkins (*8*) along lines laid down by Spenser and Fano (*21*). Briefly, the method consists in expanding the directed flux $\phi_\gamma(E_\gamma, \vec{\Omega}, r)$ in a series the terms of which are products of Laguerre and Legendre polynomials. The directed flux is integrated with respect to all directions to remove the angular dependency, then multiplied by the distance from the origin raised to integral powers. The product is integrated over all space to express the spatial dependence in terms of the "moments" of the flux, which are then used to evaluate the arbitrary coefficients in the original series in the usual way by application of the boundary conditions and utilization of the orthogonal properties of the Legendre polynomial. Having constructed the flux $\phi_\gamma(E_\gamma', r)$ due to a monoenergetic unit point source (1 photon/sec. of energy E_γ') at r', Goldstein then computes the fraction of the energy E_γ' released at r. This of course is identical with $F(E_\gamma', r' \rightarrow r)$ defined above. The results are reported in terms of an energy absorption build-up factor $B_a(E_0, \mu_0 r)$, where E_0 is E_γ' in the notation used here, μ_0 is $\mu_t(E_\gamma')$, r is $|r' - r|$, and B_a is defined in such a way that

$$F(E_\gamma', r' \rightarrow r) = \frac{B_a(E_\gamma'; |\, r' - r\,|)}{4\pi(|\, r' - r\,|)^2} \mu_e(E_\gamma') \cdot \exp\left[-\mu_t(E_\gamma')(|\, r' - r\,|)\right] \quad (29)$$

Here μ_e is the energy absorption coefficient defined by Equation (19). Values of B_a as a function of E_γ' and $|r' - r|$ are tabulated for a variety of materials by Goldstein (*8*). Taylor (*22*) has proposed an empirical correlation exponential in $|r' - r|$ which is conveniently applied to problems in gamma heating in pressure shells and thermal shields (*1*).

Conceivably the function $F(E_\gamma', |r' - r|)$ could be evaluated experimentally. Small radiation sources of various energies could be moved about in a mock-up of the system under study and the heat liberation at various points of interest measured. If a direct thermal measurement is made, the measuring instrument would have to be extremely sensitive or else the radiation-source strength would have to be inconveniently large.

The mathematical difficulties in estimating gamma heating may be considerably reduced, albeit at an unknown cost in accuracy, if one is willing to utilize the straight-ahead-scattering approximation, investigated carefully by Hurwitz, et al. (*13*). The assumption is made that photons suffer energy degradations but that the scattered photons fall into one of two classes: (*a*) photons scattered through such small angles that the deviations from the line of flight may be ignored and (*b*) photons scattered through such large angles that the resulting low-energy photons have little penetrating power (high μ_t) and are absorbed near the point of scattering. These assumptions depend upon the following arguments: (*a*) the energy distribution of the scattered photons shows a strong forward component, (*b*) the angular distribution also shows a strong forward component, (*c*) distances of interest in heat-generation calculations are measured in relatively few attenuation lengths from the primary source (and the probability

TABLE 2. PRIMARY ENERGY SOURCES

Source	Energy, mev./fission	Spectral distribution	Ref.
Fission fragments	168 ± 5	———	11
Fission-fragment beta decay	7	———	23
Fast neutrons	5	$P(E_n) = 0.484 (\sinh \sqrt{2E_n}) \exp(-E_n)$	18
Prompt fission gammas	7.5	$P(E_\gamma) = \exp(-E_\gamma)$	6
Fission-fragment gamma decay	6	Fragmentary data	4, 16, 25
Capture gammas	varies, ~7	Fragmentary data	15
Total	200		

that a photon will suffer many scatterings is small), and (*d*) photoelectric absorption increases strongly with decreasing photon energy. For example, of the photons scattered out a beam of 6 mev. photons, those scattered through angles of less than 50° carry 90% of the energy of all the scattered photons.

The approximation is applied by breaking the gamma-source spectrum into a number of energy groups. The group having the highest energy E_1 is attenuated between r' and r by the simple exponential factor $\exp[-\mu_t(E_1')(|r' - r|)]$. The next group loses photons in proportion to $\mu_t(E_2')$ but gains photons from group 1 by Compton scattering; group 3 receives photons from both groups 1 and 2; and so on. The transfer of photons between groups is a function of the energy-widths of the groups and their separation in energy and is computed from the Klein-Nishina relation [Equation (14)]. The differential equations expressing the photon "economy" for each group are integrated successively, giving the flux at r.

Hurwitz, using this multigroup technique, has calculated the energy release at r due to an energy-distributed source at r' having a typical spectrum and found that the results could be expressed in terms of an average "effective" gamma-absorption coefficient $\bar{\mu}_{eff}$, obtained by averaging μ_{eff} over the spectrum of the source.

$$\bar{\mu}_{eff} = \int_{0.5}^{\infty} \mu_{eff}(E_\gamma')P(E_\gamma')\, dE_\gamma' \quad (30)$$

where $P(E_\gamma')$ is the spectral distribution and where

$$\mu_{eff}(E_\gamma) = \mu_e(E_\gamma) + \mu_c'(E_\gamma)$$

Here μ_c' is a term added to the energy-deposition coefficient μ_e to account for the energy release due to gamma photons scattered to energies below 0.5 mev., which are assumed to be absorbed in the near vicinity of their scattering. It is evaluated by integrating the scattered-energy ratio E_γ'/E_γ given by Equation (12) with respect to the angular probability given by Equation (14) as follows:

$$\mu_c'(E_\gamma) = \int_{\theta=0}^{\theta(E_\gamma'=0.5)} (E_\gamma'/E_\gamma)P(\theta)\, d\theta \quad (31)$$

By use of the present notation Hurwitz's results may be put in the form

$$G_\gamma(r) = \int_{V_\gamma} \frac{S_{p\gamma}(r')\, dV_\gamma'\, \bar{E}_\gamma \bar{\mu}_{eff}}{4\pi(|r' - r|)^2} \cdot \exp[-\bar{\mu}_{eff}(|r' - r|)] \quad (32)$$

Primary Gamma Sources

The fission gamma and fission-fragment-decay gamma sources are both proportional to the fission density in stationary fuel reactors.

$$S_{p\gamma}(r') = N_\gamma \int_0^{\infty} \phi_n(E_n, r')\Sigma_f(E_n, r')\, dE_n \quad (33)$$

where N_γ is the number of photons, fission and decay, emitted per fission. Gamble (*6*) reports that 7.5 prompt gamma photons are released per fission, and that these have an average energy of about 1 mev. each. Their spectral distribution is approximately exponential.

$$P(E_\gamma) = \exp(-E_\gamma); \quad 0.1 < E_\gamma < 7 \text{ mev.} \quad (34)$$

This relation does not hold at the extremes of the energy range; however, the contribution from photons with energy above 7 mev. is negligible, while gammas emitted with energies less than 0.1 mev. carry less than 1% of the total energy.

The fission-fragment-decay gamma spectrum could be constructed from a knowledge of the fission yields of the various nuclides, their decay schemes, and the energies of the decay gammas. Moteff (*16*) and Clark (*4*) have tabulated the available data. Moteff listed some nuclides which emit gammas with energies down to 0.04 mev., with half-lives in excess of 30 sec. Clark extended the listing but did not tabulate gammas having energies below 0.1 mev. The sum of the gamma energies of all the nuclides reported was computed by Clark to be a little greater than 1 mev./fission. Since it has been estimated by Way (*23*) on theoretical grounds that the total fission-fragment gamma-decay energy is approximately 6 mev., it appears that there is about 4 to 5 mev. of energy/fission released by gamma emitters having half-lives shorter than 30 sec. The intensity and spectral distribution of the gammas from these short-lived isotopes is currently being investigated at Oak Ridge National Laboratory (*25*).

Day (*5*) has recently tabulated the available data on the cross sections for inelastic scattering of neutrons in various media and the spectra of the resulting gamma photons. Grace et al. (*9*) report some measurements on the scattering of 2.5-mev. neutrons in various materials. Other papers relating to the inelastic scattering of neutrons are listed by Day.

Mittelman (*15*) has tabulated the information available on the spectrum of capture gamma rays as of October, 1953. The strength of this gamma source is proportional to the local neutron flux and to the capture coefficient Σ_c. Equation (33) may be used by replacing Σ_f by Σ_c.

SUMMARY

The local rates of heat release in nuclear reactors have been formulated in terms of the local neutron and gamma fluxes. Certain special methods of estimating these fluxes were indicated and their application to the heating problem was discussed briefly. The data available on neutron- and gamma-source strengths have been reviewed.

There are four and possibly five important sources of heat the distribution of which is directly dependent on the neutron flux distribution. These are (1) fission-fragment energy, (2) fission-fragment beta-decay energy, (3) elastic collisions of fast neutrons, (4) beta decay of nuclei activated by neutron absorption, and (5) inelastic collisions of fast neutrons. Of these, sources 1 and 2 predominate in the fuel region of the heterogeneous reactors; 4 is small but may be appreciable; and 3 and 5 are negligible. In the moderator regions of such reactors, 1, 2, and 5 are negligible, and only source 3 and gamma heating need be considered. In core tanks composed of materials having high atomic numbers, gamma heating will predominate, but source 4 and possibly 5 may be appreciable. This rule applies also to thermal shields and pressure shells. Reflectors composed of materials of low atomic weight may be treated like the moderator and, if the atomic number is high, like a core tank.

Of the sources listed, sources 3 and 5 depend strongly on the distribution of the fast flux; the others may be computed to good approximations from the thermal flux densities.

The gamma-dependent heating rates are conveniently classified according to the source of the gamma rays: (1) prompt fission gammas, (2) fission-fragment-decay gammas, (3) capture gammas, (4) activated-nuclei-decay gammas, and (5) gammas resulting from inelastic scattering of fast neutrons.

In the fuel regions of heterogeneous reactors, heating due to gammas may be appreciable but is small compared with fission-fragment heating. Heating due to class 4 gammas may become appreciable if large amounts of U^{238} or Th^{232} or other material having high cross section for thermal neutrons are present. In the moderator regions of heterogeneous reactors, heating due to

classes 1 and 2 predominates, but contributions from class 3 may be appreciable in regions adjacent to coolants and structural materials having appreciable capture cross sections. In core tanks composed of materials having low capture cross sections, classes 1 and 2 predominate, but 5 may be appreciable, as in zirconium. In iron thermal shields and pressure shells classes 1 and 2 tend to predominate near the inner surface, class 3 at intermediate depths, while class 5 predominates at greater depths (*26*). Class 4 follows class 3 in distribution, but is much less important and can generally be neglected.

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NOTATION

B_a = energy-absorption build-up factor, defined by Equation (29), and tabulated in reference *8*.

D = neutron-diffusion coefficient, reference *7*

E = kinetic energy of particle, mev.

E_f = average kinetic energy of fission fragments plus average energy of betas resulting from decay of fission fragments, 187 mev./fission

E_s = energy of nascent neutrons

$f(r)$ = geometric form of neutron-flux-density distributions.

$f(E_\gamma', R)$ = fraction of energy of photon emitted at a point with energy E_γ' that is dissipated in an annular sphere of radius R and thickness dR.

$F(E_\gamma', r' \rightarrow r)$ fraction of energy of photons emitted at r' with energy E_γ' that is dissipated at r per unit volume about r

G = specific rate of heat release due to dissipation of the energy of nuclear radiations, energy units per unit volume per unit time

I_0 = zero-order modified Bessel function of the first kind

m = atomic mass of neutron, $\sim$1 atomic mass units

M = atomic mass of nucleus in atomic mass units

(mc^2) = rest mass of electron. The value in mev. is about 0.51.

n = number of neutrons per unit volume

N_i = number of β particles of characteristic energy $E_{\beta i}$ emitted in decay chain following a neutron capture

N_γ = number of photons (prompt and decay) emitted per fission

$p(E_s, E_n)$ = capture-escape probability

$P(\theta)$ = probability that a neutron or photon will be scattered through an angle θ per unit angle about θ

$P(E_s)$ = spectral distribution of fission neutrons, reference *18*.

$P(E_\gamma, r')$ = probability that a photon having energy E_γ will be emitted or scattered at r', per unit volume about r'

$P(r', r)$ = probability that a photon emitted or scattered at r' will travel in the direction of r per stearadian about that direction

$P(r' \rightarrow r)$ = probability that a photon or neutron at r' traveling in the direction of r will arrive at r without having suffered a collision

$P(E_\gamma)$ = spectral distribution of gamma flux density or the fraction of photons having energy E_γ per unit energy range about E_γ

Q = rate of heat release in nuclear reactor, energy units per unit time

r = radius vector

S = source function, particles generated per unit volume per unit time

th = thermal energy, $\sim$0.025 ev.

V = volume throughout which particle sources are distributed

Greek

α = physical property of nucleus defined by Equation (6)

δ = average fraction of energy of incident particle that is released locally during a collison

θ = angle of deflection in a scattering collision

μ = photon interaction coefficient, probability of a collision of specified type occurring per centimeter of travel, events per photon-centimeter

$\bar{\mu}_{eff}$ = mean effective gamma-absorption coefficient, defined by Equation (30)

μ_e = photon energy-absorption coefficient defined by Equation (19)

μ_t = photon-absorption coefficient defined by Equation (25)

ξ = logarithmic energy decrement, $\xi \cong 2/(M + \frac{2}{3})$ (*See* reference *7*.)

ρ = radial coordinate in cylindrical geometry

Σ = macroscopic cross section, probability of a collision of specified type occurring per centimeter of travel, events per neutron-centimeter

τ = Fermi age, sq. cm.

ϕ = flux density, number of centimeters traversed by neutrons or photons per cubic centimeter per second

∇^2 = Laplacian operator

Subscripts and Superscripts

a = absorption

c = Compton scattering of photons or capture of neutrons

e = elastic scattering of neutrons (*See also* μ_e.)

f = fission (*Also see* E_f.)

ff = fission fragments

$ff\beta$ = beta particles resulting from beta decay of fission fragments

i = inelastic scattering

n = neutrons

N = nucleus

pe = photoelectric

pp = pair production

$p\gamma$ = primary gamma particles

t = total

β = beta particle

γ = gamma particle

$'$ = prime, denotes postcollision condition or energy

$-$ = superbar, denotes average value of variable

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